

Simple Kinetic Theory of Dilute Gases



Equilibrium vs Non-Equilibrium

- Until now we have focused entirely on systems in equilibrium.
- Even though it may not seem so, equilibrium is much easier to deal with than non-equilibrium.
- To deal with non-equilibrium requires a detailed understanding of motion at the microscopic level.

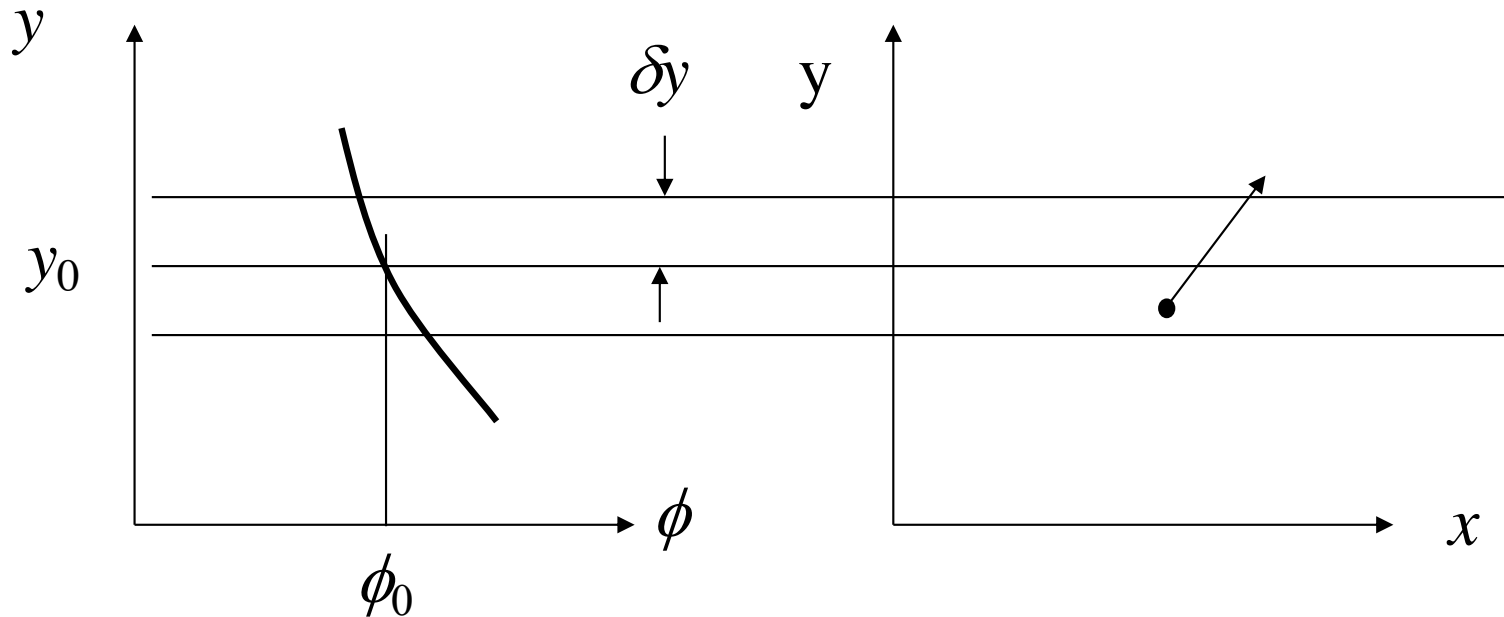
What are the physics?

- Molecules are constantly moving.
- Molecules carry properties such as identity, mass, momentum and energy which they acquire by colliding with other molecules.
- At equilibrium the net flow of molecular properties must be zero.
- But if there is a gradient of a molecular property, then there can be a net flux of that property.
 - i.e. u_x, u_y, u_z, n_i, T, P



Transport

Consider a property ϕ that is some mean molecular quantity per molecule that varies with y



When a molecule crosses y_0 it carries, or transports a value of ϕ that is characteristic of where it had its last collision. Because of the gradient, the flux of ϕ will be different in the two directions leading to a net flux of the property.



Fluxes

- The values of the property carried past y_0 in the positive and negative directions are

$$\phi(y_0 - \delta y) \quad \text{and} \quad \phi(y_0 + \delta y)$$

- What is the total flux of molecules carrying this property across y_0 , and what is the appropriate value of δy to use?
- The mean free path seems like a good choice.

Molecular Flux

- The flux of molecules in one direction is (where c_i is the component of velocity in the i -th direction.)

$$J_{i_{molecules}} = \int c_i f(c_i) dc_i$$

- The flux of some property ϕ carried by the flow of molecules in one direction is

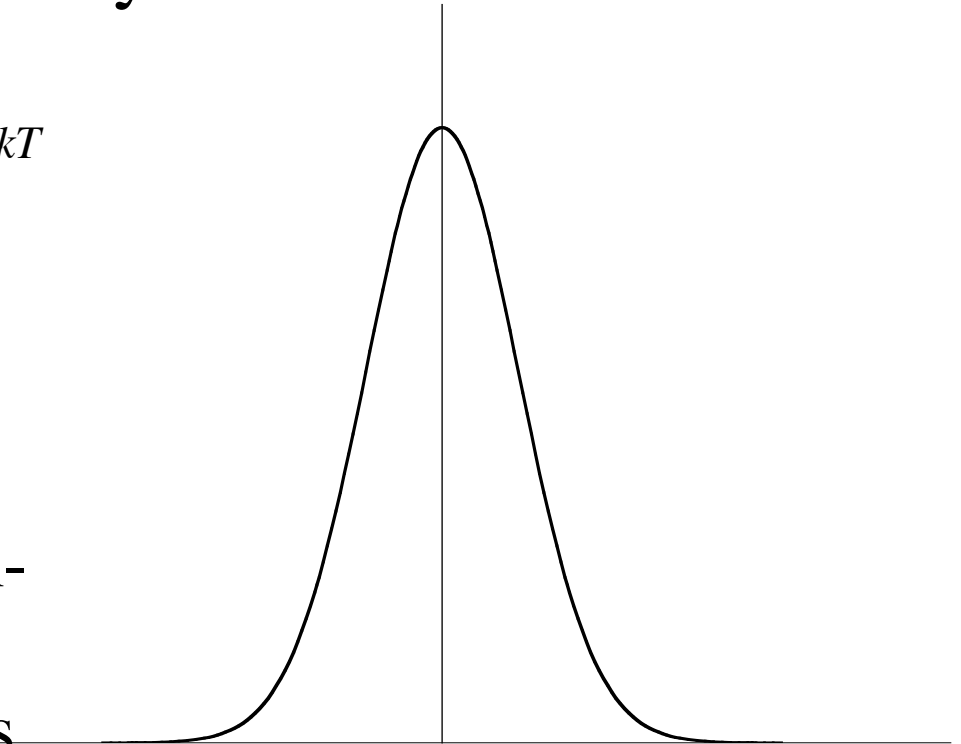
$$J_{i_\phi} = \int \phi c_i f(c_i) dc_i$$

The Velocity Distribution Function

From thermodynamics we have shown that at equilibrium molecular velocity is distributed as:

$$f(c_i) = \left(\frac{m}{2\pi kT} \right)^{1/2} e^{-mc_i^2 / 2kT}$$

where c_i is the component of velocity in the i -th direction. It turns out that under mildly non-equilibrium conditions, the velocity distribution function is close to the Maxwellian.



The Maxwellian



Molecular Flux

- Thus the molecular flux is

$$J_{\text{molecules}} = \frac{n\bar{c}}{4}$$

- And the flux of some property

$$J_{\phi} = \phi \frac{n\bar{c}}{4}$$

- Average speed

$$\bar{c} = \sqrt{\frac{8kT}{\pi m}}$$



Net Transport

- Assume, then, that

$$\delta y = \alpha l$$

- Where l is the mean free path and α is a constant of order unity
- Thus, net flux of property ϕ becomes

$$J_{\phi} = \frac{n\bar{c}}{4}(\phi(y_0 - \alpha l) - \phi(y_0 + \alpha l))$$



Net Transport

- Expanding ϕ in a Taylor series

$$\phi(y_0 + \delta y) = \phi(y_0) + \frac{d\phi}{dy} \delta y + \dots$$

- And

$$\phi(y_0 - \delta y) = \phi(y_0) - \frac{d\phi}{dy} \delta y + \dots$$

- And keeping the 1st order terms

$$J_{\phi} = -\frac{\alpha}{2} n \bar{c} l \frac{d\phi}{dy}$$

